

# Geometric phase outside a Schwarzschild black hole and the Hawking effect

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## Abstract

We study the Hawking effect in terms of the geometric phase acquired by a two-level atom as a result of coupling to vacuum fluctuations outside a Schwarzschild black hole in a gedanken experiment. We treat the atom in interaction with a bath of fluctuating quantized massless scalar fields as an open quantum system, whose dynamics is governed by a master equation obtained by tracing over the field degrees of freedom. The nonunitary effects of this system are examined by analyzing the geometric phase for the Boulware, Unruh and Hartle-Hawking vacua respectively. We find, for all the three cases, that the geometric phase of the atom turns out to be affected by the space-time curvature which backscatters the vacuum field modes. In both the Unruh and Hartle-Hawking vacua, the geometric phase exhibits similar behaviors as if there were thermal radiation at the Hawking temperature from the black hole. So, a measurement of the change of the geometric phase as opposed to that in a flat space-time can in principle reveal the existence of the Hawking radiation.

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## I. INTRODUCTION

In quantum sense, a black hole is not completely black, but emits thermal radiation with a black body spectrum. This is the well-known Hawking radiation which is first found by Hawking in the paradigm of quantum field theory in curved space-time [1]. Ever since Hawking's original work, this striking effect has attracted a great deal of interest, and extensive work has been done trying to re-derive and understand it in various different contexts [1–10], ranging from Euclidean quantum gravity [2] to quantum entanglement [10].

In this paper, we will try to understand, in the framework of open quantum systems, the Hawking effect by studying the geometric phase acquired by a static two-level atom due to its coupling to the vacuum fluctuations outside a Schwarzschild black hole in a gedanken experiment. The geometric phase is an important concept in quantum theory. In 1984, Berry showed that when the Hamiltonian of a closed quantum system evolves adiabatically in a cyclic way, the state of the system acquires an additional phase besides the familiar dynamical one, which has a purely geometric origin [11]. Berry's work was soon generalized to nonadiabatic [12] and noncyclic evolution [13]. The geometric phase has been extensively studied both theoretically and experimentally [14]. In recent years, it has been demonstrated that geometric phase may have many potential applications, for instance, fault-tolerant quantum computation [15]. However, due to the inevitable interactions between the qubits and the environment, a pure state will be driven to a mixed one. Therefore, the study of geometric phase of systems under nonunitary evolutions becomes important, and some work has been done to generalize the geometric phase to open quantum systems [16–20], and the effects of different kinds of decoherence sources on the geometric phase have been analyzed [21–25]. By studying the geometric phase of an open system, we can draw some information about the nonunitary evolution of the system and the characteristics of the environment the system coupled to. In principle, the geometric phase factor can be measured by interfering the system which has undergone the above evolution with one that does not. Noteworthily, a new type of experiment on the quantum geometric phase of an open system undergoing nonunitary evolution has recently been done, where the phase is determined by measuring the decoherence factor of the off-diagonal elements of the reduced density matrix of the system [26].

Recently, Martin-Martinez et al. [27] have studied the geometric phase of an accelerated

atom which couples to a single-mode of a scalar field in vacuum and found that the phase differs from that of an inertial one as a direct consequence of the Unruh effect. Thus, in principle, an interferometry experiment can be performed to reveal the effects of acceleration. Later, we have considered a more realistic case, i.e., an accelerated two-level system which couples to all vacuum modes of electromagnetic (rather than scalar) fields in a realistic multipolar coupling scheme, and proposed using the geometric phase of non-unitary evolution to detect the Unruh effect [28]. In the current paper, we study the geometric phase acquired by a two-level atom interacting with a bath of fluctuating quantized massless scalar fields in vacuum outside a Schwarzschild black hole in a gedanken experiment, in the hope of gaining understanding of the Hawking effect in a new perspective. In the gedanken experiment, we first prepare the atom in a pure state, adiabatically put it at a fixed radial distance outside the black hole, then the atom interacts with the fluctuating scalar fields in vacuum, and as a result, it evolves from a pure state to a mixed state. During this nonunitary evolution, a geometric phase is acquired. The atom in the gedanken experiment can be thought of as being held static by an imaginary string with tension, or the buoyancy of the Hawking radiation, or even some other unknown mechanism. The reduced dynamics of the atom is studied by tracing over the field degrees of freedom from the total system. The Boulware, Unruh and Hartle-Hawking vacua will be considered. At this point, let us note that the theory of open quantum system has been fruitfully applied to understand the Unruh [29], Hawking [9], and Gibbons-Hawking [30] effects respectively in a way that is different from the traditional one.

## II. THE MASTER EQUATION

The Hamiltonian of the whole system takes the form

$$H = H_s + H_\phi + H' . \quad (1)$$

Here  $H_s$  is the Hamiltonian of the atom, and its general form is taken to be

$$H_s = \frac{\omega_0}{2} \sum_{i=1}^3 n_i \sigma_i , \quad (2)$$

in which  $\sigma_i$  is the Pauli matrix, and  $\omega_0$  is the energy level spacing of the atom. For simplicity, we can write it as  $H_s = \frac{1}{2}\omega_0\sigma_3$ .  $H_\phi$  is the Hamiltonian of the scalar field, whose explicit

expression is not required here. We couple the atom and the scalar field by analogy with the electric dipole interaction [31]

$$H' = \mu \sigma_2 \Phi(t, \mathbf{x}) , \quad (3)$$

where  $\mu$  is the coupling constant, which we assume to be small.

The initial state of the whole system is characterized by the total density matrix  $\rho_{tot} = \rho(0) \otimes |0\rangle\langle 0|$ , in which  $\rho(0)$  is the initial reduced density matrix of the atom, and  $|0\rangle$  is the vacuum state of the field. In the frame of the atom, the evolution in the proper time  $\tau$  of the total density matrix  $\rho_{tot}$  satisfies

$$\frac{\partial \rho_{tot}(\tau)}{\partial \tau} = -\frac{i}{\hbar} [H, \rho_{tot}(\tau)] . \quad (4)$$

The evolution of the reduced density matrix  $\rho(\tau)$  can, in the limit of weak coupling, be written in the Kossakowski-Lindblad form [29, 32–34]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -\frac{i}{\hbar} [H_{\text{eff}}, \rho(\tau)] + \mathcal{L}[\rho(\tau)] , \quad (5)$$

where

$$\mathcal{L}[\rho] = \frac{1}{2} \sum_{i,j=1}^3 a_{ij} [2 \sigma_j \rho \sigma_i - \sigma_i \sigma_j \rho - \rho \sigma_i \sigma_j] . \quad (6)$$

The matrix  $a_{ij}$  and the effective Hamiltonian  $H_{\text{eff}}$  are determined by the Fourier and Hilbert transforms of the correlation functions

$$G^+(x - y) = \mu^2 \langle 0 | \Phi(x) \Phi(y) | 0 \rangle , \quad (7)$$

which are defined as follows

$$\mathcal{G}(\lambda) = \int_{-\infty}^{\infty} d\tau e^{i\lambda\tau} G^+(\tau) , \quad (8)$$

$$\mathcal{K}(\lambda) = \frac{P}{\pi i} \int_{-\infty}^{\infty} d\omega \frac{\mathcal{G}(\omega)}{\omega - \lambda} , \quad (9)$$

in which  $P$  denotes principal value. Then the coefficients of the Kossakowski matrix  $a_{ij}$  can be expressed as [29]

$$a_{ij} = A \delta_{ij} - iB \epsilon_{ijk} n_k + C n_i n_j , \quad (10)$$

where

$$A = \frac{1}{4} [\mathcal{G}(\omega_0) + \mathcal{G}(-\omega_0)] , \quad B = \frac{1}{4} [\mathcal{G}(\omega_0) - \mathcal{G}(-\omega_0)] , \quad C = -A . \quad (11)$$

The effective Hamiltonian  $H_{\text{eff}}$  contains a correction term, the so-called Lamb shift, and one can show that it can be obtained by replacing  $\omega_0$  in  $H_s$  with a renormalized energy level spacing  $\Omega$  as follows [29]

$$H_{\text{eff}} = \frac{1}{2}\hbar\Omega\sigma_3 = \frac{\hbar}{2}\{\omega_0 + \frac{i}{2}[\mathcal{K}(-\omega_0) - \mathcal{K}(\omega_0)]\}\sigma_3 . \quad (12)$$

For convenience, the density matrix  $\rho$  is expressed in terms of the Pauli matrices,

$$\rho(\tau) = \frac{1}{2}\left(1 + \sum_{i=1}^3 \rho_i(\tau)\sigma_i\right) . \quad (13)$$

Taking Eq. (13) into Eq. (5), one can show that the components of the Bloch vector  $|\rho(\tau)\rangle$  satisfy

$$\frac{\partial}{\partial\tau}|\rho(\tau)\rangle = -2\mathcal{H}|\rho(\tau)\rangle + |\eta\rangle , \quad (14)$$

where  $|\eta\rangle = \{0, 0, -4B\}$ , and  $\mathcal{H}$  takes the form

$$\mathcal{H} = \begin{pmatrix} 2A + C & \Omega/2 & 0 \\ -\Omega/2 & 2A + C & 0 \\ 0 & 0 & 2A \end{pmatrix} . \quad (15)$$

Now we assume that the initial state of the atom is  $|\psi(0)\rangle = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle$ , and then it evolves from a pure state to a mixed state due to interaction with the external environment, namely, a bath of fluctuating vacuum scalar fields. The time-dependent reduced density matrix of the atom can be easily worked out as

$$\rho(\tau) = \begin{pmatrix} e^{-4A\tau} \cos^2\frac{\theta}{2} + \frac{B-A}{2A}(e^{-4A\tau} - 1) & \frac{1}{2}e^{-2(2A+C)\tau - i\Omega\tau} \sin\theta \\ \frac{1}{2}e^{-2(2A+C)\tau + i\Omega\tau} \sin\theta & 1 - e^{-4A\tau} \cos^2\frac{\theta}{2} - \frac{B-A}{2A}(e^{-4A\tau} - 1) \end{pmatrix} . \quad (16)$$

### III. GEOMETRIC PHASE IN OPEN TWO-LEVEL SYSTEM

In this section, we use the master equation governing the reduced dynamics of the atom in the last section to calculate the geometric phase of a static atom in the background of a Schwarzschild space-time, which is described by the following metric

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1 - 2M/r} - r^2(d\theta^2 + \sin^2\theta d\phi^2) . \quad (17)$$

When a curved space-time is concerned, a delicate issue is how to determine the vacuum state of the quantum fields. Normally, a vacuum state is associated with non-occupation

of positive frequency modes. However, the positive frequency of field modes is defined with respect to the time coordinate. Therefore, to define positive frequency, one has to first specify a definition of time. In a spherically symmetric black hole background, three different vacuum states, i.e., the Boulware [35], Unruh [36], and Hartle-Hawking [37] vacuum states, have been defined, each corresponding to a different choice of time coordinate. In the following, we examine the geometric phase of the atoms in all these vacua.

### A. The Boulware vacuum

Let us begin our discussion with the Boulware vacuum, which is defined by requiring normal modes to be positive frequency with respect to the Killing vector  $\partial/\partial t$ . In order to analyze the geometric phase of the system, we make use of the master equation Eq. (5) and see how the phase is related to it.

The geometric phase for a mixed state under nonunitary evolution is defined as [19]

$$\gamma = \arg \left( \sum_{k=1}^N \sqrt{\lambda_k(0)\lambda_k(T)} \langle \phi_k(0) | \phi_k(T) \rangle e^{-\int_0^T \langle \phi_k(\tau) | \dot{\phi}_k(\tau) \rangle d\tau} \right), \quad (18)$$

where  $\lambda_k(\tau)$  and  $|\phi_k(\tau)\rangle$  are the eigenvalues and eigenvectors of the reduced density matrix  $\rho(\tau)$ . In order to get the geometric phase, we first calculate the eigenvalues of the density matrix (16) to find  $\lambda_{\pm}(\tau) = \frac{1}{2}(1 \pm \eta)$ , in which  $\eta = \sqrt{\rho_3^2 + e^{-4(2A+C)\tau} \sin^2 \theta}$  and  $\rho_3 = e^{-4A\tau} \cos \theta + \frac{B}{A}(e^{-4A\tau} - 1)$ . It is obvious that  $\lambda_-(0) = 0$ . As a result, the only contribution comes from the eigenvector corresponding to  $\lambda_+$

$$|\phi_+(\tau)\rangle = \sin \frac{\theta_\tau}{2} |+\rangle + \cos \frac{\theta_\tau}{2} e^{i\Omega\tau} |-\rangle, \quad (19)$$

where

$$\tan \frac{\theta_\tau}{2} = \sqrt{\frac{\eta + \rho_3}{\eta - \rho_3}}. \quad (20)$$

The geometric phase can be calculated directly using Eq. (18)

$$\gamma = -\Omega \int_0^T \cos^2 \frac{\theta_\tau}{2} d\tau. \quad (21)$$

Plugging the explicit form of  $\theta_\tau$ ,  $\eta$  and  $\rho_3$  into Eq. (21), we have

$$\gamma = - \int_0^T \frac{1}{2} \left( 1 - \frac{R - R e^{4A\tau} + \cos \theta}{\sqrt{e^{-4C\tau} \sin^2 \theta + (R - R e^{4A\tau} + \cos \theta)^2}} \right) \Omega d\tau. \quad (22)$$

The result of this integral is rather tedious. Here, for small  $\gamma_0/\omega_0$ , we perform a series expansion of the geometric phase for a single quasi-cycle and obtain to the first order [28]

$$\gamma \approx -\pi(1 - \cos \theta) + \frac{2\pi^2}{\omega_0}(2B - C \cos \theta) \sin^2 \theta . \quad (23)$$

It is obvious that the geometric phase of the system is characterized by the coefficients  $B$  and  $C$  which determine the Kossakowski matrix  $a_{ij}$ , and we now calculate them explicitly.

Let us now calculate the geometric phase of a two-level atom which is held static at a fixed radial distance by some additional force, an imaginary string with tension, for instance, outside a Schwarzschild black hole. The trajectory can be described by

$$t(\tau) = \frac{1}{\sqrt{g_{00}}}(\tau - \tau_0), \quad r(\tau) = r_0, \quad \theta(\tau) = \theta_0, \quad \phi(\tau) = \phi_0 . \quad (24)$$

In the Boulware vacuum, the Wightman function for the scalar field on the trajectory Eq. (24) is given by [38–40]

$$G^+(x, x') = \mu^2 \sum_{lm} \int_0^\infty \frac{e^{-i\omega\Delta t}}{4\pi\omega} |Y_{lm}(\theta, \phi)|^2 [|\vec{R}_l(\omega, r)|^2 + |\overleftarrow{R}_l(\omega, r)|^2] d\omega . \quad (25)$$

The Fourier transform with respect to the proper time is

$$\mathcal{G}(\lambda) = \int_{-\infty}^\infty e^{i\lambda\tau} G^+(x, x') d\tau = \mu^2 \sum_l \frac{2l+1}{8\pi\lambda} [|\vec{R}_l(\lambda\sqrt{g_{00}}, r)|^2 + |\overleftarrow{R}_l(\lambda\sqrt{g_{00}}, r)|^2] \theta(\lambda) , \quad (26)$$

where  $\theta(\lambda)$  is the step function, and we have used the relation

$$\sum_{m=-l}^l |Y_{lm}(\theta, \phi)|^2 = \frac{2l+1}{4\pi} . \quad (27)$$

The above Fourier transform is hard to evaluate, since we do not know the exact form of the radial functions  $\vec{R}_l(\omega, r)$  and  $\overleftarrow{R}_l(\omega, r)$ . Here, we choose to compute it both close to the event horizon and at infinity, with the help of the properties of the radial functions in asymptotic regions [40]:

$$\sum_{l=0}^\infty (2l+1) |\vec{R}_l(\omega, r)|^2 \sim \begin{cases} \frac{4\omega^2}{1 - \frac{2M}{r}} , & r \rightarrow 2M , \\ \frac{1}{r^2} \sum_{l=0}^\infty (2l+1) |B_l(\omega)|^2 , & r \rightarrow \infty , \end{cases} \quad (28)$$

$$\sum_{l=0}^\infty (2l+1) |\overleftarrow{R}_l(\omega, r)|^2 \sim \begin{cases} \frac{1}{4M^2} \sum_{l=0}^\infty (2l+1) |B_l(\omega)|^2 , & r \rightarrow 2M , \\ \frac{4\omega^2}{1 - \frac{2M}{r}} , & r \rightarrow \infty . \end{cases} \quad (29)$$

Plugging Eq. (26) into Eq (11), and using Eq. (28) and (29), we have

$$A = B = -C \approx \frac{\gamma_0}{4} [1 + f(\omega_0 \sqrt{g_{00}}, r)] , \quad (30)$$

in which  $f(\omega, r)$  is defined as

$$f(\omega, r) = \frac{1}{4r^2\omega^2} \sum_{l=0}^{\infty} (2l+1) |B_l(\omega)|^2 , \quad (31)$$

and  $\gamma_0 = \mu^2\omega_0/2\pi$  is the spontaneous emission rate. So, after some straightforward calculations, we obtain, in the asymptotic regions,

$$\gamma \approx \begin{cases} -\pi(1-\cos\theta) + \pi^2 \frac{\gamma_0}{2\omega_0} \sin^2\theta (2+\cos\theta) (1 + g_{00} f(\omega_0 \sqrt{g_{00}}, 2M)) & r \rightarrow 2M , \\ -\pi(1-\cos\theta) + \pi^2 \frac{\gamma_0}{2\omega_0} \sin^2\theta (2+\cos\theta) (1 + g_{00} f(\omega_0 \sqrt{g_{00}}, r)) & r \rightarrow \infty . \end{cases} \quad (32)$$

The result above shows that, after a single quasi-cycle, the static atom acquires a geometric phase, which depends on the geometric properties of its state space. The first term in the above equation is the one we would have obtained if the system were isolated from the environment, and the second term is the correction induced by the interaction between the atom and the environment. It is interesting to note that, in both asymptotic regions, the correction of the geometric phase takes the same form, which is what one expects in the Minkowski vacuum[24, 28] multiplied by a factor  $(1 + g_{00} f(\omega_0 \sqrt{g_{00}}, r))$ . The additional correction as opposed to the Minkowski vacuum, which is proportional to  $g_{00} f(\omega_0 \sqrt{g_{00}}, r)$ , can be attributed to the backscattering of the vacuum field modes off the space-time curvature of the black hole. This is similar in a sense to the case of the reflection of the field modes at a reflecting boundary in a flat space-time. From the definition of the grey-body factor  $f(\omega, r)$  (31) we can see that, in the limit of  $r \rightarrow \infty$ ,  $g_{00} f(\omega_0 \sqrt{g_{00}}, r)$  tends to be zero, and the result returns to the case in the Minkowski vacuum, which means that the Boulware vacuum corresponds to our familiar concept of a vacuum state at large radii.

## B. The Unruh vacuum

Now let us go on our discussion to the Unruh vacuum, which is supposed to be the vacuum state best approximating the state following the gravitational collapse of a massive body. The Wightman function in the Unruh vacuum is given by [38–40]

$$G^+(x, x') = \mu^2 \sum_{ml} \int_{-\infty}^{\infty} \frac{e^{-i\omega\Delta t}}{4\pi\omega} |Y_{lm}(\theta, \phi)|^2 \left[ \frac{|\vec{R}_l(\omega, r)|^2}{1 - e^{-2\pi\omega/\kappa}} + \theta(\omega) |\overleftarrow{R}_l(\omega, r)|^2 \right] d\omega , \quad (33)$$

where  $\kappa = 1/4M$  is the surface gravity of the black hole. Its Fourier transform is

$$\begin{aligned}\mathcal{G}(\lambda) &= \int_{-\infty}^{\infty} e^{i\lambda\tau} G^+(x, x') d\tau \\ &= \frac{\mu^2}{8\pi\lambda} \sum_{l=0}^{\infty} \left[ \theta(\lambda\sqrt{g_{00}})(1+2l) |\overleftarrow{R}_l(\lambda\sqrt{g_{00}}, r)|^2 + \frac{(1+2l)|\overrightarrow{R}_l(\lambda\sqrt{g_{00}}, r)|^2}{1-e^{-2\pi\lambda\sqrt{g_{00}}/\kappa}} \right],\end{aligned}\quad (34)$$

in which  $\kappa_r$  is defined as  $\kappa/\sqrt{g_{00}}$ . Inserting Eq. (34) into Eq. (11), one finds that

$$r \rightarrow 2M : \begin{cases} A = -C \approx \frac{\gamma_0}{4} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, 2M) + \frac{2}{e^{2\pi\omega_0/\kappa_r} - 1}], \\ B \approx \frac{\gamma_0}{4} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, 2M)], \end{cases}\quad (35)$$

and

$$r \rightarrow \infty : \begin{cases} A = -C \approx \frac{\gamma_0}{4} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, r) + \frac{2}{e^{2\pi\omega_0/\kappa_r} - 1} g_{00} f(\omega_0\sqrt{g_{00}}, r)], \\ B \approx \frac{\gamma_0}{4} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, r)]. \end{cases}\quad (36)$$

Straightforward calculations lead to

$$\gamma \approx \begin{cases} -\pi(1-\cos\theta) + \pi^2 \frac{\gamma_0}{2\omega_0} \sin^2\theta \times \\ \left[ (2+\cos\theta)(1+g_{00} f(\omega_0\sqrt{g_{00}}, 2M)) + \frac{2}{e^{2\pi\omega_0/\kappa_r} - 1} \cos\theta \right], & r \rightarrow 2M \\ -\pi(1-\cos\theta) + \pi^2 \frac{\gamma_0}{2\omega_0} \sin^2\theta \times \\ \left[ (2+\cos\theta)(1+g_{00} f(\omega_0\sqrt{g_{00}}, r)) + g_{00} f(\omega_0\sqrt{g_{00}}, r) \frac{2}{e^{2\pi\omega_0/\kappa_r} - 1} \cos\theta \right]. & r \rightarrow \infty \end{cases}\quad (37)$$

From the equations above, we can see that, in the vicinity of the event horizon, the first term in the environment induced geometric phase is the same as the one in the Boulware vacuum, and the second one is proportional to a Planckian factor. At infinity, the Planckian factor is modified by a grey-body factor  $g_{00} f(\omega_0\sqrt{g_{00}}, r)$ , which is caused by the backscattering off the space-time curvature. In order to have a better understanding of the features of the geometric phase and of what it tells us about, it would be rewarding to examine here the geometric phase of a static atom in a thermal bath at temperature  $1/\beta$  in a flat space-time. In this case, the Wightman function reads

$$G^+(x, x') = -\frac{1}{4\pi^2} \sum_{m=-\infty}^{\infty} \frac{1}{(t-t'-im\beta-i\varepsilon)^2 - |\mathbf{x}-\mathbf{x}'|^2},\quad (38)$$

and similar calculations leads to

$$\gamma \approx -\pi(1-\cos\theta) + \pi^2 \frac{\gamma_0}{2\omega_0} \sin^2\theta \left( 2 + \cos\theta + \frac{2}{e^{\beta\omega_0} - 1} \cos\theta \right).\quad (39)$$

Comparing Eq. (37) and Eq. (39), one finds that, apart from the backscattering of the space-time curvature, the result is the same as that in the case of a two-atom system immersed in a thermal bath at an effective temperature  $T = \kappa_r/2\pi = T_H/\sqrt{g_{00}}$  in a flat space-time, where  $T_H = \kappa/2\pi$  is the Hawking temperature [1]. This clearly suggests that at the horizon, there is a thermal flux going outwards. However, when  $r \rightarrow 2M$ , the temperature  $T$  is divergent, since this effective temperature is a joint effect of both the thermal flux from the black hole and the Unruh effect due to that the system is accelerating with respect to the local free-falling inertial frame so as to maintain at a fixed distance from the black hole, and the acceleration diverges at the horizon. As the atoms are placed farther away from the black hole, the thermal radiation becomes weaker due to the backscattering off the space-time curvature. At the infinity, the grey-body factor vanishes and it returns to the Minkowski vacuum case. This suggests that, in the Unruh vacuum, no thermal radiation is felt at the infinity due to the backscattering of the outgoing thermal radiation off the space-time curvature.

### C. The Hartle-Hawking vacuum

Finally, let us come to the Hartle-Hawking vacuum. The Wightman function in this vacuum is given by [38–40]

$$G^+(x, x') = \mu^2 \sum_{ml} \int_{-\infty}^{\infty} \frac{|Y_{lm}(\theta, \phi)|^2}{4\pi\omega} \left[ \frac{e^{-i\omega\Delta t}}{1 - e^{-2\pi\omega/\kappa}} |\vec{R}_l(\omega, r)|^2 + \frac{e^{i\omega\Delta t}}{e^{2\pi\omega/\kappa} - 1} |\overleftarrow{R}_l(\omega, r)|^2 \right] d\omega ,$$

and its Fourier transform is

$$\mathcal{G}(\lambda) = \int_{-\infty}^{\infty} e^{i\lambda\tau} G^+(x, x') d\tau = \mu^2 \sum_{l=0}^{\infty} \frac{(1+2l)}{8\pi\lambda} \left[ \frac{|\vec{R}_l(\lambda\sqrt{g_{00}}, r)|^2}{1 - e^{-2\pi\lambda/\kappa_r}} + \frac{|\overleftarrow{R}_l(-\lambda\sqrt{g_{00}}, r)|^2}{1 - e^{-2\pi\lambda/\kappa_r}} \right] . \quad (40)$$

Similar calculations then lead to

$$r \rightarrow 2M : \begin{cases} A = -C \approx \frac{\gamma_0}{4} \frac{e^{2\pi\omega_0/\kappa_r} + 1}{e^{2\pi\omega_0/\kappa_r} - 1} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, 2M)] , \\ B \approx \frac{\gamma_0}{4} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, 2M)] , \end{cases} \quad (41)$$

$$r \rightarrow \infty : \begin{cases} A = -C \approx \frac{\gamma_0}{4} \frac{e^{2\pi\omega_0/\kappa_r} + 1}{e^{2\pi\omega_0/\kappa_r} - 1} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, r)] , \\ B \approx \frac{\gamma_0}{4} [1 + g_{00} f(\omega_0\sqrt{g_{00}}, r)] , \end{cases} \quad (42)$$

and

$$\gamma \approx \begin{cases} -\pi(1-\cos\theta) + \pi^2 \frac{\gamma_0}{2\omega_0} \sin^2\theta \times \\ \left[ (2+\cos\theta)(1+g_{00} f(\omega_0\sqrt{g_{00}}, 2M)) + \frac{2}{e^{2\pi\omega_0/\kappa_r} - 1} (1+f(\omega_0\sqrt{g_{00}}, 2M)) \cos\theta \right], & r \rightarrow 2M \\ -\pi(1-\cos\theta) + \pi^2 \frac{\gamma_0}{2\omega_0} \sin^2\theta \times \\ \left[ (2+\cos\theta)(1+g_{00} f(\omega_0\sqrt{g_{00}}, r)) + \frac{2}{e^{2\pi\omega_0/\kappa_r} - 1} (1+f(\omega_0\sqrt{g_{00}}, r)) \cos\theta \right]. & r \rightarrow \infty \end{cases} \quad (43)$$

Here, unlike that in the Unruh vacuum case, the geometric phase takes the same form in two asymptotic regions. The first term is the same as that in the Boulware vacuum, and the second term is composed by two parts, one is proportional to the standard Planckian factor and the other is proportional to a Planckian factor modified by a grey-body factor caused by the backscattering off the space-time curvature. This suggests that there are thermal radiation outgoing from the horizon and that incoming from infinity, both of which are weakened by the backscattering off the curvature on their way. In both asymptotic regions, the impact of the black hole to the geometric phase is the same as that of a thermal bath at temperature  $T = T_H/\sqrt{g_{00}}$  in a flat space-time. As  $r \rightarrow \infty$ , the effective temperature becomes the Hawking temperature, since the acceleration needed to maintain the two-level atom system at a fixed radial distance vanishes, and the temperature is purely due to the thermal bath the black hole is immersed in. Therefore, the Hartle-Hawking vacuum does not correspond to our familiar concept of vacuum. It is actually a state that describes a black hole in equilibrium with an infinite sea of blackbody radiation [40].

#### IV. CONCLUSION

We have studied the evolution of a two-level atom interacting with a bath of fluctuating quantum scalar fields at a fixed radial distance outside a Schwarzschild black hole in the framework of open quantum systems, and calculated the geometric phase of the atom in the Boulware, Unruh and Hartle-Hawking vacua respectively. We find, for all the three cases, that the geometric phase of the atom turns out to be affected by space-time curvature which backscatters the vacuum field modes. In the Unruh vacuum, the geometric phase behaves as

if there were an outgoing flux of thermal radiation which is backscattered by the space-time curvature. In the Hartle-Hawking vacuum, the geometric phase exhibits behaviors as if the atom were in a thermal bath at an effective temperature which reduces to the Hawking temperature at infinity.

Generally speaking, the geometric phase is of significance only if we compare the situation of the atom to some other situations. For our current case, it is natural to compare the atom kept at a fixed position to the one released free. However, the calculation of the geometric phase of a free-falling atom, although of great physical importance, is technically much more difficult than the situation considered in our current paper. The reason is that in a Schwarzschild space-time, the vacuum fluctuations, namely, the environment the atom is coupled to, varies with the radial distance from the black hole. We will leave this to a future investigation. Nevertheless, as it has already been done in a recent experiment [26], we may manage to measure the decoherence factor of the off-diagonal elements of the reduced density matrix directly in order to get the geometric phase. So, we can obtain the geometric phase in a Schwarzschild space-time and that in a flat space-time respectively, and then compare the results in the two situations to reveal the effects of Hawking radiation. In summary, our analysis shows, at least in principle, that geometric phase might provide a potentially new way to detect the Hawking radiation, albeit difficulties of constructing specific experimental procedures that can actually be carried out.

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